

A weakly universal cellular automaton with 2 states on the tiling $\{11, 3\}$

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Abstract

In this paper, we construct a weakly universal cellular automaton with two states only on the tiling $\{11, 3\}$. The cellular automaton is rotation invariant and it is a true planar one.

1 Introduction

The paper makes use of the improvements introduced in the paper [7]. As most of the author's papers in this topic, this paper makes use of the railway model, see [10, 5]. We just remind the reader that the circuit simulates a register machine instead of a Turing machine as in [10]. The Euclidean circuit consists in tracks which are either straight segments or quarters of a circle. The circuit allows crossings and it makes use of three kinds of switches: the fixed one, the flip-flop and the memory switch. A unique locomotive runs over the circuit and the evolution of the positions of the switches in time allows us to simulate a computation. Although initially devised for the Euclidean plane, this model can be implemented in the tessellations of the hyperbolic plane, especially in the tessellations $\{p, 4\}$ and $\{p+2, 3\}$ which are spanned by a tree which, in each case allows a rather easy way to implement horizontals and verticals.

The first implementation, in 2002, was performed in the pentagrid, the tiling $\{5, 4\}$ of the hyperbolic plane, see [1]. It required 22 states. The number of states was lowered down to 9 in the same tiling in 2008, see [8]. At the same time, the circuit was implemented in the heptagrid, the tiling $\{7, 3\}$ of the hyperbolic plane, see [9], requiring 6 states. Both papers of 2008 gave the same implementation. The smaller number of states is compensated by a higher number of neighbours. A bit later, I reduced the number of states down to 4 in the heptagrid by a slight change in the locomotive: replacing the previously green front cell by a blue one, the same colour as that of the milestones. This smaller number of states in the heptagrid compared to the pentagrid is not surprising. It was again observed a bit later in 2012 with the implementation in the tiling $\{13, 3\}$ of a weakly universal rotation invariant and planar cellular automaton with two states only, see [6, 5]. This latter implementation introduces many features which allowed to lower the number of states. The first idea was to mark the tracks by milestones instead of assigning a specific colour to the tracks. Then, the idea was to allow one-way tracks only. This implies a strong change in the switches. The fixed one is simplified to a passive switch only: the switch is only needed when the locomotive comes from one of the two tracks which join into a single one at the switch. The flip-flop was already a one-way switch in its original implementations as it must be crossed actively only. The constraint of one-way tracks implies to split the memory switch into two switches: an active one and a passive one. The working of the switch makes it necessary to connect the active switch to the passive one: when the locomotive comes from the non-selected track in the passive switch, the selection changes there and it also must change in the active switch.

These new features were enough to reduce the number of states down to two of them in the dodecagrid, the tiling $\{5, 3, 4\}$ of the hyperbolic $3D$ -space. The space allows us to get rid of the crossings, which is not at all the case in the plane. Moreover, the introduction of one-way tracks makes the number of crossings to seriously increase: a previous two-way crossing has to be implemented by four one-way crossings. The implementation in $\{13, 3\}$ introduced a new feature coming from roadway traffic, the round-about. It also reduced the locomotive to a single cell, making it closer to a particle. Trying to implement these new ideas in the heptagrid, I recently obtained a weakly universal cellular automaton with three states only, see [7]. Now, in this paper, I had to improve the implementation of the one-way switches. In particular, introducing patterns used in asynchronous cellular automata, I could organize the implementation of the flip-flop and the active-memory switch in a similar way, using two new and simpler components, the fork and

the selector. The difference in the switches is now given by the assembling of these new components, a bit different in these two switches. I thought it could be useful to implement these new elements in order to obtain a cellular automaton with two states in a grid $\{p, 3\}$ with $p < 13$. I could do that for $p = 11$ by replacing the passive memory switch by a combination of the fork with a new structure, the controller, a bit simpler than the passive memory switch itself.

We make all of this more precise in Section 2.

Before turning to Section 2, let us explain how we shall illustrate our implementation. The tiling $\{11, 3\}$ in the Poincaré's disc model is illustrated by the leftmost picture of Figure 1.

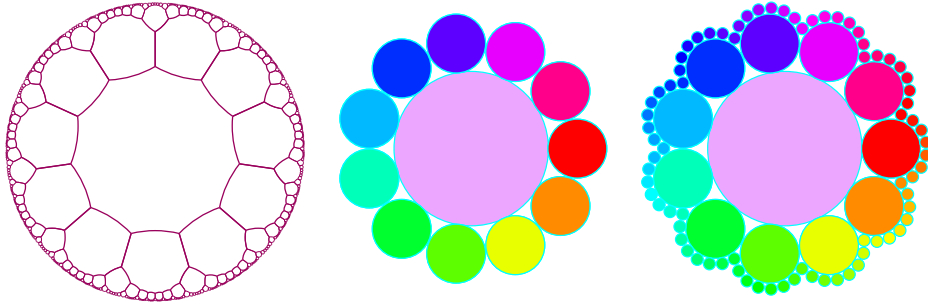


Figure 1 *To left: the tiling $\{11, 3\}$ in the Poincaré's model. To right: another representation of a cell in the tiling $\{11, 3\}$ and the same one together with its neighbours.*

We do not see very much in this representation, so that we shall replace it by the two other illustrations given in Figure 1. As will be seen in the further illustrations, this representation allows us to give it some flexibility which will improve the readability of the figures.

2 The scenario

In this section, we precisely describe the implementation of the railway circuit. Here, we admit that it is possible to devise such a circuit that the motion of the locomotive simulates the computation of a register machine thanks to the positions of all switches of the circuit. This is explained with all details in [5].

Let us remind the reader that the systematic one-way track organization of the circuit leads to a more complex representation of the memory switch. On the left-hand side of Figure 2 we have a sketchy representation of the

two-way memory switch. We remind the reader that the two-way switch may be crossed either passively or actively. On the right-hand side, we have the one-way switch. It consists in two one-way half-switches: an active half-switch on the left-hand side and a passive one on the right-hand side. A connection goes from the passive half-switch to the active half-switch: this is needed when the selection has to be changed. It first changes at the passive part which detects that the locomotive came from the non-selected track, and the necessity to change the selection is passed to the active part through the orange path of Figure 2.

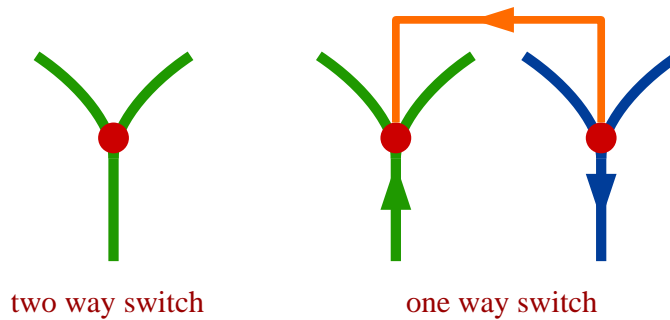


Figure 2 *Comparison between the two-way and the one-way representation of the memory switch.*

In this section, we precisely explain the new features in Sub-section 2.1 and how they are implemented in Sub-section 2.2.

2.1 General patterns

Let us first list the elements we shall study in Sub-section 2.2. With each name of an element, we sketchy describe what the element is expected to perform.

First, the **elements of the tracks**: each cell of the track is marked with appropriate milestones. Such an element may be crossed either by a single locomotive or by two locomotives running together, contiguously. We shall see the elements in Sub-section 2.2.

Then, the second structure we need is the **fixed switch**. As mentioned in the introduction, for one-way tracks it is a passive structure only. It gathers two tracks which are melted into a single one after the cell at which the two tracks arrive.

Next, we have two patterns involved by the round-about: the **duplicator** and the **selector**.

The duplicator has two points of connection with the tracks: an **entrance** and an **exit**. The locomotive arrives through the entrance. Two locomotives leave the duplicator through the **exit**.

The selector has three points of connection with the tracks: an **entrance** and two **exits**. Each exit correspond to the number of locomotives entering the selector. If a single locomotive enters, it leaves the selector through **exit 1** which is connected with a track leaving the round-about. If two contiguous locomotives enter the selector, one of them leaves the selector through **exit 2**. Exit 2 is attached to a piece of tracks leading to another selector.

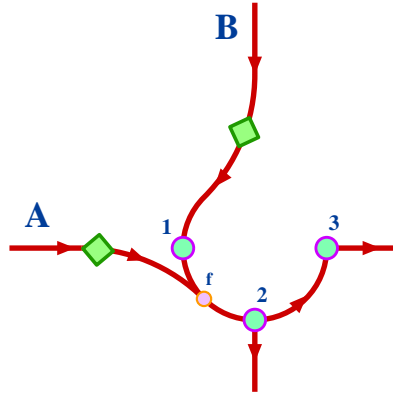


Figure 3 *The round-about. The arrows indicate the path used by the locomotive.*

This explains how a **round-about** works. This is illustrated by Figure 3. In the figure, the duplicator is illustrated by a green rhomboid pattern. The selectors are illustrated by a disc. A round-bout assembles two duplicators, three selectors and a fixed switch, denoted by **f** in the figure. The locomotive crosses one disc and leaves the round-about at the second one. When it arrives through **A**, **B**, the locomotive is transformed into two contiguous locomotives after crossing the duplicator. When it meets the first selector, one locomotive is killed while the second one goes on its way on the round-about, towards the second selector. At the second selector, 3, 2 respectively, it leaves the round-about to continue its way on the same tracks.

Next, we consider the **controller**, a structure which is used by both the flip-flop and the active memory switch. The controller has two states : **accept** or **reject**. If on **accept**, the locomotive entering the controller is accepted and it is allowed to cross it in order to go on its way on the tracks. If the controller is on **reject**, then the locomotive is not accepted: it does

not leave the pattern, it is killed there.

Next we consider the **fork**. It is different from the duplicator: here too, one locomotive enters and two ones exit, but the two exiting locomotives are on different tracks. The fork has an **entrance** and two **exits**. When the locomotive has crossed the fork, there is one locomotive on each track leaving the pattern.

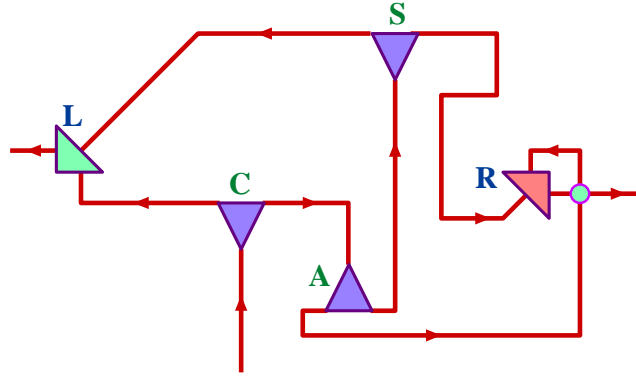


Figure 4 Association of forks and controllers in order to constitute a flip-flop.

Both structures, the controller and the fork are used to implement a flip-flop and an active memory switch. Figure 4 shows how to combine three forks and two controllers in order to obtain a flip-flop.

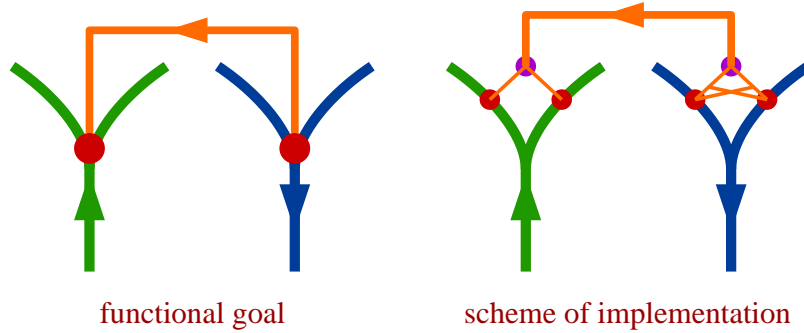


Figure 5 To left: the functional goal of the new implementation of the memory switch with one-way tracks. To right: the scheme of implementation.

Note that this is connected with what we have indicated in Figure 2. Now it is time to make this idea more precise. Figure 5 gives the general scheme of implementation of the memory switch under the constraint of

one-way tracks.

Now we turn to a more exact implementation of the scheme illustrated by the right-hand side part of Figure 5. First, Figure 6 shows how to combine the same elements in order to obtain an active memory switch, following the scheme of Figure 5.

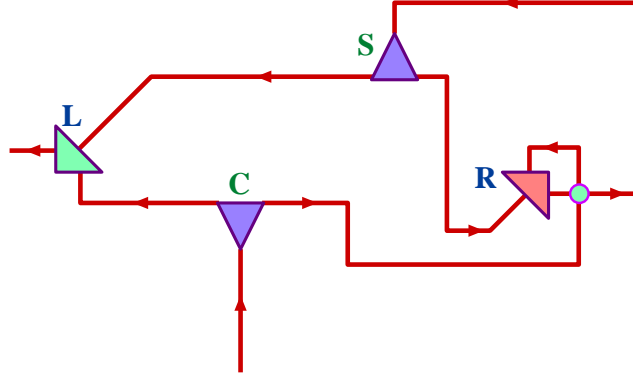


Figure 6 Association of forks and controllers in order to constitute an active memory switch.

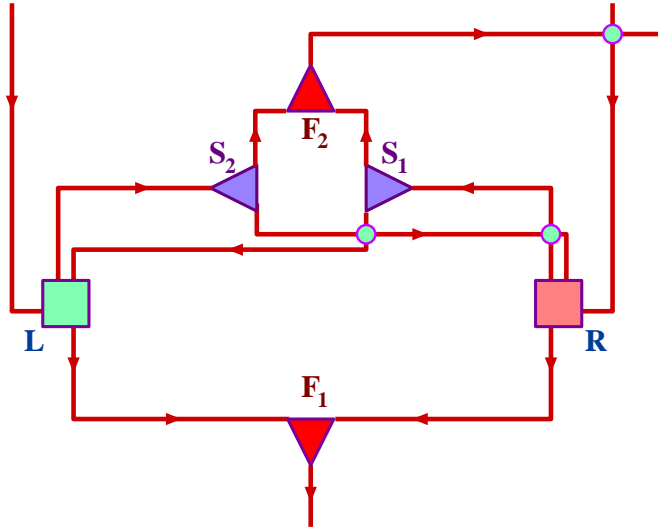


Figure 7 Association of forks and new controllers in order to constitute a passive memory switch.

In order to implement the scheme proposed by Figure 5, we have to introduce a new element. Note that the idea is to align the presentation of the passive memory switch with that of the active one by splitting a unique control at the switch into two independent controls on each branch of the

tracks going to the switch. Now, due to the basic difference of working of the switches, we cannot use the same controller as in the active memory switch. The reason is that here, the controller must not stop the locomotive which crosses its structure. It has only to react to such a passage in case it is not the selected one. This means that the controller performs a double function, but not at the same time. When the controller is on the presently non-selected track, it changes the selection and, at the same time, it sends a signal to the active memory switch in order to change the selection there too.

Figure 7 illustrates how the fixed switch, the fork and the new controller have to be associated in order to implement a passive memory switch, following the scheme of implementation given in Figure 5. Note that the track leaving the switch F_2 in Figure 7 is the track arriving to the fork S in Figure 6.

2.2 Detailed implementations

Now, let us turn to the exact implementation of the patterns described in Sub-section 2.1. We shall intensively use the representation introduced by Figure 1. Now, as can be noticed already from Figure 9, we shall not always represent the neighbours of the same cell by circles or coloured discs of the same size. The size of a neighbour will mainly be dictated by its role in the considered configuration. In particular, the necessity to represent the neighbourhood of this neighbour will also play a role.

We shall successively study the elements of the tracks, the duplicating structure, the selector, then the fork, the controller for the active switches and the controlling structure of the passive memory switch. We shall study **idle configurations** only, *i.e.* configurations when the locomotive is not in the neighbourhood of the structure. We shall illustrate the motion of the locomotive through the structures when we shall establish the rules, see Section 3.

The tracks

The tracks are the first object we have to implement. We must not neglect this point. First of all, without tracks, the information of what happens at some switch will for ever remain in the switch, which is of no use for the computation. Second, the tracks certainly constitute the biggest part of the circuit in term of quantity of involved elements. Transported into the hyperbolic plane, the horizontal parts of the tracks represented in Figures 4

to 7 require a huge amount of cells.

Using Figure 8 of the present sub-subsection, we can see on Figure 9 how we can implement a track going from a cell to any other one. Note that in this figure, we make use of the elements of the lower row in Figure 8. In principle, we could make use of these elements only. Indeed, considering two fixed non-neighbouring cells P and Q . Consider a shortest path from the tile supporting P to that which supports Q . Figure 8 how to organize a track joining P to Q , using these elements only. Now, we use also the elements of the first row of Figure 8 in order to make the implementation of the other structures easier. In particular this will be used in the case of the round-about.

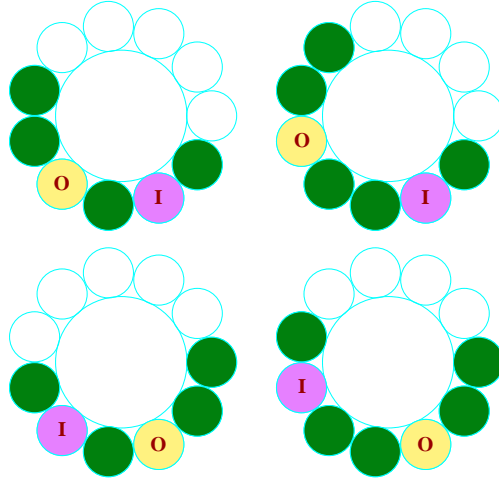


Figure 8 *The four possible elementary elements of the tracks.*

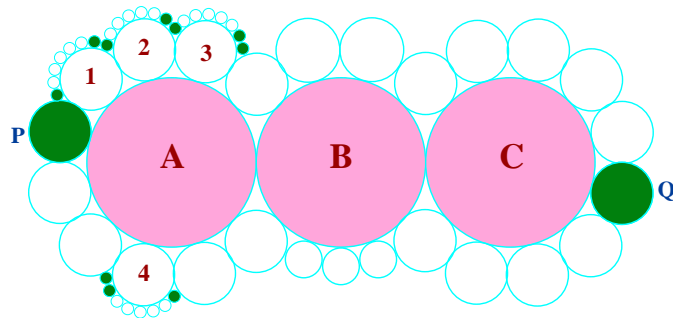


Figure 9 *The use of elements of the tracks in order to define a track going from the cell P to the cell Q .*

Note that in Figure 8, the first column deals with cells of the track which turn around a fixed black cell: call it the **pivot** of the cell. But, as we can see on Figure 9, it is needed to consider the case when the tracks go from one pivot to another one. This case is dealt with by the second column of Figure 8. For such cells, we say that it is in between two pivots.

Next, Figure 10 illustrates the implementation of the passive fixed switch. Note the particular configuration of the entrances to the switch. They are elements of the track but their working is a bit different as will appear in Section 3.

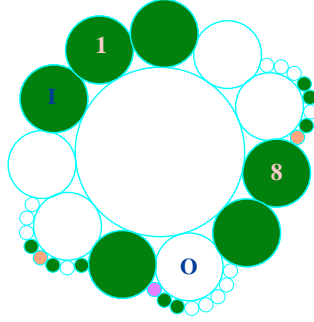


Figure 10 *The idle configuration of the fixed switch.*

The round about

In this sub-subsection, we successively study the duplicator and the selector.

The idle configuration of the duplicator is illustrated by Figure 11. From Sub-section 2.1, we know that a single locomotive enters the structure and that two contiguous ones leave it.

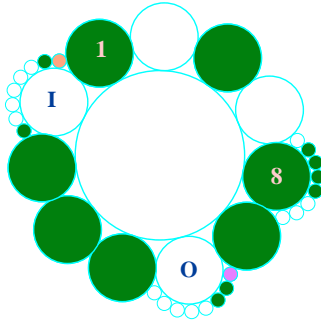


Figure 11 *The idle configuration of the duplicator in a round-about.*

The single locomotive enters through the cell marked by *I* in the figure,

while the two locomotives successively leave through the cell marked by O . The presence of the locomotive makes cell 8 flash by turning to white and then, at the next time, by turning back to black. When cell 8 is white, the main cell remains to be black which creates the needed second locomotive while the first one leaves the main cell. After that, cell 8 returns to black so that the second locomotive also leave the duplicator.

After the duplicator, we now look at the selector. It is a more complex structure. It has to count how many locomotives arrive at the device and then, depending on whether it is the case of one or two locomotives, it reacts in different ways.

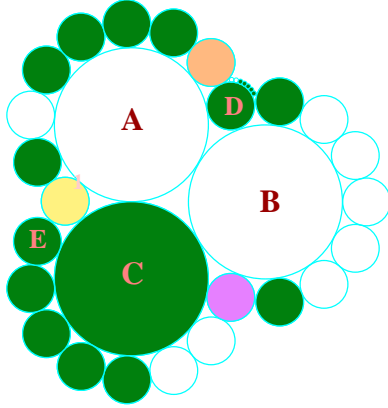


Figure 12 *The idle configuration of the selector used by a round-about.*

The locomotive arrive to the structure by a the common neighbour of B and C at the bottom of B in Figure 12. Then, the locomotive arrives to A . There, the locomotive is duplicated on the two exits from A : the exit which goes along E and the one which goes along D , see the figure. Two cells can see both A and B : the cells C and D . This allows the structure to count how many locomotives arrive at it. We know that this number is one or two so that, writing the state of A and then that of B , the configuration seen of AB seen from C and D is bw in the case of one locomotive and bb in the case of two of them. If two locomotive arrive, one is killed, this means that A is black for one time only and D turns to white while C remains black. The effect of this action is that the locomotive which arrives close to D is killed while that which was created at the common neighbour between C and E goes on along the track which will lead it to the next selector of the round-about. When a single locomotive arrives, so that the configuration seen from C and D is bw , C turns to white and D remains black. Accordingly,

the locomotive created at the neighbour of D goes on its way, leaving the round-about while that which was created close to C and E is killed.

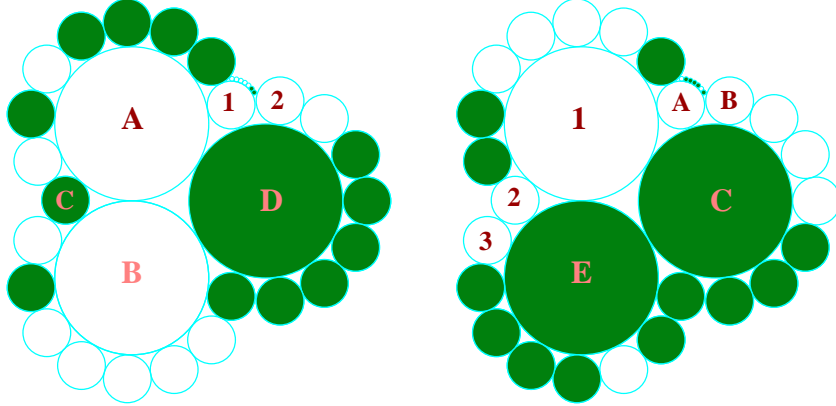


Figure 13 *Zooming at the idle configuration of the selector.*

Figure 13 zooms at D and C which allows us to see more clearly how the crossing is processed. Note that E plays a role: the locomotive created at 1, right-hand side of the figure arrives at 2 when C becomes white. So that E has to become white when C is flashing. This allows us to kill the locomotive sent in this direction.

The active switches

From the structure of the selector, we can easily derive a structure which we call the **fork** which receives one locomotive and dispatches a copy of it two different directions.

For the convenience of the reader, we reproduce the picture in the left-hand side part of Figure 14. We remind the reader that this structure is used in the flip-flop, the active memory switch and in the passive memory switch too, see Figures 4, 6 and 7. In the right-hand side part of the same figure, we illustrate the controlling device used by both the flip-flop and the active memory switch, again look at Figures 4 and 6.

In the right-hand side part of Figure 14, we zoom at the cell **c** which looks at the passage of the locomotive. If the cell is white, the locomotive is allowed to pass and, necessarily, it passes: in this case, the cell **t** which is on the track followed by the locomotive has the neighbourhood of an element of the track. If the cell is black, then it prevents the locomotive from entering the cell **t** so that the locomotive is killed.

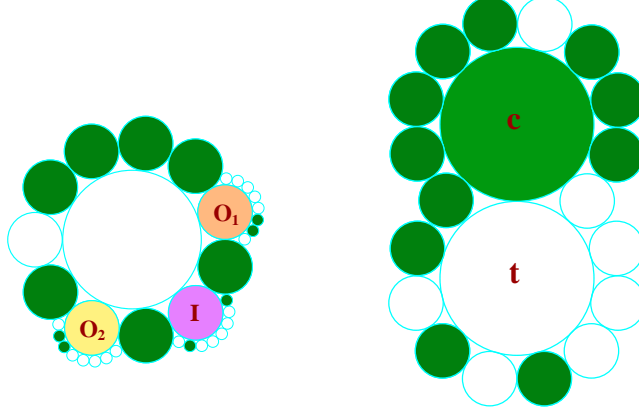


Figure 14 *The idle configuration of the fork and of the controller used in the active switches.*

The passive memory switch

From Subsection 2.1, we know that the passive memory switch requires a more complex structure than the passive one: here we have rather a **sensor** than a controller. The reason is that the sensor does not stop the locomotive.

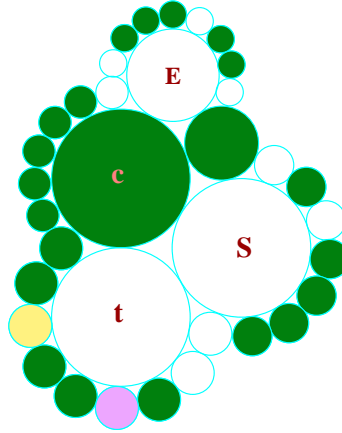


Figure 15 *The sensor used by the passive memory switch.*

However, the sensor is a more active structure: if it is black and if a locomotive passes through the cell **t**, the cell **S** of the sensor which can see both **c** and **t** realizes that the locomotive is passively running on the non-

selected track. So that the sensor changes the selection: **c** becomes white. But on the other sensor of the switch, the cell **c** is also white so that second cell **c** must become black. This is why the cell **S** sends a locomotive which reaches both the second cell **c** and also the fork of the active switch in order to change the states in both its controllers.

Now, the signal sent by one of the sensors to the other enters the cell **E** which turns the white state of the cell **c** to black.

3 Checking the automaton: the rules

In this section, we give the rules used by the automaton and we prove them. We first describe the format used for the rules and then we shall provide the rules used for each configuration. We illustrate the rules by figures describing the motion of the locomotive in the different situations described in Section 2 and, especially, in Subsection 2.2. The rules are numbered, which will allow us to follow there application in the scheduling tables of the section. These tables provides us with the state of the key cells in the circuit at different times together with the rule which was applied at that time for this cell.

First, we fix the format of the rules. To this purpose, we number the sides of the cell and we say that the side **i** is shared by the cell and by its neighbour **i**. We shall consider that $\mathbf{i} \in \{1..11\}$ and that the numbers increase while counter-clockwise turning around a cell. As here we construct a rotation invariant automaton, it is not important to fix which side is side **1**. For instance, considering Figure 8, the rule applied to the cell of the track for the leftmost cell in the upper row will be denoted WBBWBWBWWWWW. In this format, in the non underlined part of the word, the i^{th} letter from the left indicates the state of the neighbour **i**: this can easily be checked on Figure 8. This format will be that of the rules which will be displayed from now on. In the word denoting a rule, the non-underlined part is called the **context** of the rule: it is the list of the sates of the neighbours, from 1 to 11.

Second: before listing the rules we have to note that the role of many rules consists in keeping a considered configuration persistent. By this we mean that after a possible modification introduced by the passage of the locomotive, the configuration recovers a state it keeps most of the time or, at least, for a long time. As an example of the second situation we have the controllers of the flip-flop: the indication of whether the passage by the locomotive is allowed or not depends on the selected track which may be changed but, once the selection is fixed, the indication remains permanent

until a possibly new one is fixed, much later if we consider the number of steps of the automaton. In many cases, the configurations are marked by black cells, B in the rules while the background of the space is white, marked by W in the rules. In most situations, the white cells of the background have at most two contiguous neighbours. So that the first ten rules of the automaton are:

Table 1 *The first rules: conservative rules for the milestones.*

1 <u>WWWWWWWWWWWW</u>	5 <u>WBBWWWWWWWW</u>	9 <u>WBBBWWWWWWWW</u>
2 <u>BWWWWWWWWWWB</u>	6 <u>BWBWWWWWWWWB</u>	10 <u>BBBBWWWWWWWWB</u>
3 <u>WBWWWWWWWWWW</u>	7 <u>WBWBWWWWWWWW</u>	
4 <u>BBWWWWWWWWWB</u>	8 <u>BBWBWWWWWWWB</u>	

3.1 Motion rules

As Figure 8 represents the idle configuration only, we here provide the reader with an illustration of the motion of one or two locomotives through an element of the tracks, see Figures 16 and 17. This will allow the reader to easier follow the checking of the rules given by Table 2. It is not superfluous to remind the reader that several rules of Table 1 are also applied in this situation.

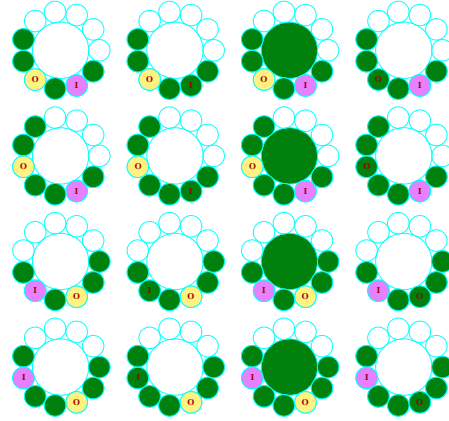


Figure 16 *The four possible motions for the elements of the tracks. Here, for a single locomotive.*

Let us look carefully at the situation. With Figure 8 in mind, fix a cell of the track. Let us consider that side **1** is the leftmost black neighbour of the cell. Then, the rules of Table 2 apply to the cell of the track. Rules 11 and 18, 28 and 34 apply to an **idle configuration**: this means that the locomotive is far away from that cell, so that if it is white, it remains white

for the next time. For the milestones of the track, we can see that in an idle configuration rules 4 applies. For neighbours of a milestone, rules 3 and 5 apply while rule 9 applies to the neighbours of the cell which are its entrance and its exit for the locomotive. Rule 10 applies to some milestones when the locomotive is in the cell.

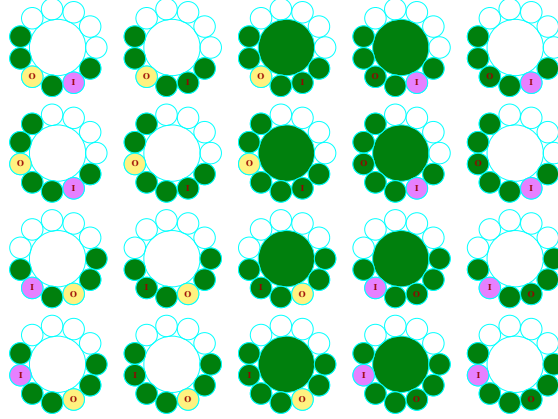


Figure 17 *The four possible motions for the elements of the tracks for two contiguous locomotives.*

First, consider a cell with a single pivot. When a single locomotive crosses it, the rules which apply to the cell are rules 11, 12, 13 and 14 in one direction, and rules 23, 24, 25 and 26 in the opposite direction. When two contiguous locomotives cross the cell, rules 12 or 24 apply as the cell see the first locomotive. When rules 12 and 24 are applied, the first locomotive occupies the cell while the second one is a neighbour of the cell, the neighbour which was occupied by the first locomotive at the previous time. This is why rules 15 and 27 apply to the first locomotive. At the next time, the second locomotive occupies the cell while the first one already left it and occupies a neighbour of the cell, neighbour 3 or 4, depending on the direction of the motion. Next, rules 16 and 28 apply to the second locomotive, making it to leave the cell. The cell can see the first locomotive in neighbour 3 or 4, depending on which type of cell, and it returns to white. Accordingly, at the next step, the cell is white and the second locomotive is in neighbour 3 or 4. As the cell has to remain white, rules 16 and 28 apply. At the next time, the configuration is idle again so that rules 11 and 23 apply to the cell.

Note that a whole train of consecutive locomotives could cross the cell using the rules for the case of two consecutive locomotives.

For a cell in between two pivots, the principle is exactly the same. The

difference is that instead of one black neighbour between the entrance and the exit, there are two consecutive black neighbours.

Table 2 *The motion rules: for a single locomotive and when two contiguous locomotives travel on the tracks. In each case, motion in one direction and motion in the opposite one.*

from left to right		from right to left	
one pivot	in between	one pivot	in between
i o	i o	o i	o i
11 <u>WBWBWBWBWBWBWB</u>	17 <u>WBWBWBWBWBWBWB</u>	23 <u>WBWBWBWBWBWBWB</u>	29 <u>WBWBWBWBWBWBWB</u>
12 <u>WBWBWBWBWBWBWB</u>	18 <u>WBWBWBWBWBWBWB</u>	24 <u>WBWBWBWBWBWBWB</u>	30 <u>WBWBWBWBWBWBWB</u>
13 <u>BWBWBWBWBWBWB</u>	19 <u>BWBWBWBWBWBWB</u>	25 <u>BWBWBWBWBWBWB</u>	31 <u>BWBWBWBWBWBWB</u>
14 <u>BWBWBWBWBWBWB</u>	20 <u>BWBWBWBWBWBWB</u>	26 <u>BWBWBWBWBWBWB</u>	32 <u>BWBWBWBWBWBWB</u>
2 locomotives	2 locomotives	2 locomotives	2 locomotives
15 <u>BWBWBWBWBWBWB</u>	21 <u>BWBWBWBWBWBWB</u>	27 <u>BWBWBWBWBWBWB</u>	33 <u>BWBWBWBWBWBWB</u>
16 <u>BWBWBWBWBWBWB</u>	22 <u>BWBWBWBWBWBWB</u>	28 <u>BWBWBWBWBWBWB</u>	34 <u>BWBWBWBWBWBWB</u>

Table 3 Motion of a single locomotive. To left: motion from left to right. To right: motion from right to left.

	1	2	3	4	5	6	7	8		1	2	3	4	5	6	7	8
1	13	12	11	17	17	11	11	14	1	26	23	23	29	29	23	24	25
2	14	13	12	17	17	11	11	11	2	23	23	23	29	29	24	25	26
3	11	14	13	18	17	11	11	11	3	23	23	23	29	30	25	26	23
4	11	11	14	19	18	11	11	11	4	23	23	23	30	31	26	23	23
5	11	11	11	20	19	12	11	11	5	23	23	24	31	32	23	23	23
6	11	11	11	17	20	13	12	11	6	23	24	25	32	29	23	23	23
7	11	11	11	17	17	14	13	12	7	24	25	26	29	29	23	23	23
8	12	11	11	17	17	11	14	13	8	25	26	23	29	29	23	23	24

Table 4 *Motion of two contiguous locomotives. To left: motion from left to right. To right: motion from right to left. As shown in the last line, cells 1 and 8 are neighbours on the tracks too.*

	1	2	3	4	5	6	7	8		1	2	3	4	5	6	7	8
1	16	15	12	17	17	11	11	14	1	26	23	23	29	29	24	27	28
2	14	16	15	18	17	11	11	11	2	23	23	23	29	30	27	28	26
3	11	14	16	21	18	11	11	11	3	23	23	23	30	33	28	26	23
4	11	11	14	22	21	12	11	11	4	23	23	24	33	34	26	23	23
5	11	11	11	20	22	15	12	11	5	23	24	27	34	32	23	23	23
6	11	11	11	17	20	16	15	12	6	24	27	28	32	29	23	23	23
7	12	11	11	17	17	14	16	15	7	27	28	26	29	29	23	23	24
8	15	12	11	17	17	11	14	16	8	28	26	23	29	29	23	24	27

Tables 3 and 4 illustrate the motion in the following way. Eight consecutive cells of the track are taken in one direction in Table 3, in the other direction in Table 4. In both cases, cells are numbered from 1 to 8 and cells 4 and 5 are in between two consecutive pivots. The tables indicate for each cell and for each time which instruction applies. When the number of the instruction is in **red bold digits**, this means that the corresponding cell is black before the rule is applied, otherwise it is white. Figure 9 allows us to

check this. Note that Tables 3 and 4 were filled up by a computer program simulating the automaton. Later on, such tables will be called **schedule tables**.

3.2 Fixed switch

The fixed switch has been described in Sub-section 2.2.

Figure 18 illustrates the motion of one and two contiguous locomotives through a fixed switch. The rules are given in Table 5. Basically, the motion is that of a locomotive through an element of the track. The first two lines illustrate the passive passage of a single locomotive. In the first row, the locomotive comes from the left, in the second row, it comes from the right. The rule for keeping the idle configuration is rule 35. Rule 36 allows the locomotive to enter the locomotive from the left. Rule 39 does the same for a locomotive coming from the right.

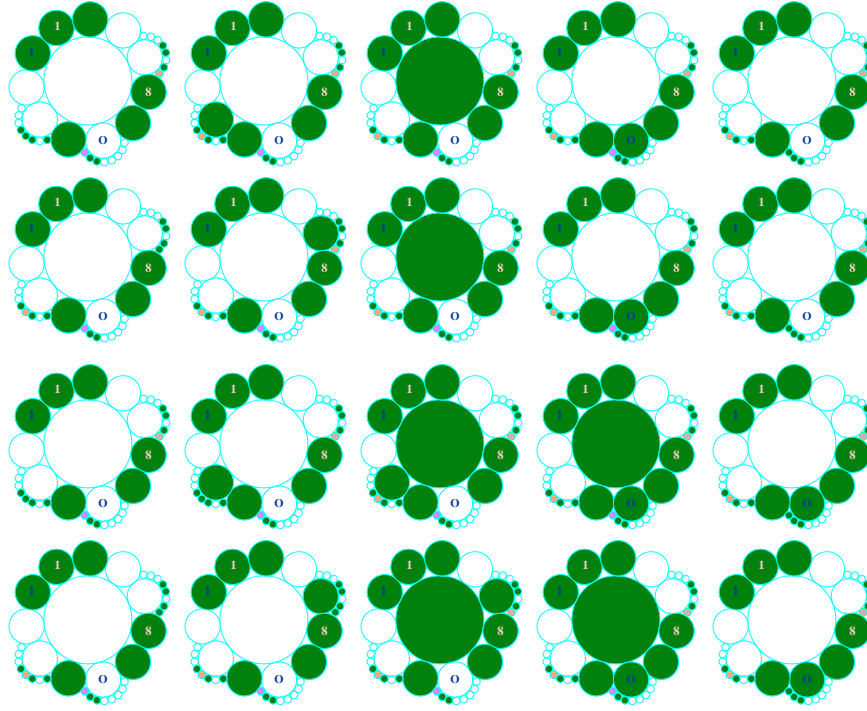


Figure 18 *The motion of the locomotive through a fixed switch. First two rows: a single locomotive. Last two rows: two contiguous locomotives.*

Table 5 *The rules which handle a fixed switch.*

a single locomotive	two locomotives	tracks again
35 i_1 o i_2	40 i_l o i_r	43
36 <u>WBWBWBWBWBWB</u>	41 <u>BBBBWBWBWBWB</u>	44 <u>BWBWBWBWBWBWB</u>
37 <u>WBWBWBWBWBWB</u>	42 <u>BBBBWBWBWBWB</u>	45 <u>BWBWBWBWBWBWB</u>
38 <u>WBWBWBWBWBWB</u>		46 <u>BWBWBWBWBWBWB</u>
39 <u>WBWBWBWBWBWB</u>		

Rules 36 and 37 make the locomotive enter and then leave the cell. Rule 44 witnesses that the locomotive leaves the cell. In the case of two locomotives, rules 40 and 42 allow the second locomotive to enter the cell while the first locomotive is about to leave it. There are two rules as there are two possible entries. Rule 41 makes the second locomotive leave the cell. Tables 6 and 7 are the schedule tables illustrating the crossing of the switch by the locomotive(s). Here, the center of the switch is denoted by \mathbb{F} , the entrances by i_ℓ for the left-hand side one and by i_r for the right-hand side one. The exit is denoted by o . For each exit/entrance cell, its neighbour of the track is indicated as v_ℓ , v_r and w , respectively. As the cells i_ℓ and i_r which are cells of the tracks are used with a different exit from what is normally achieved, we need two additional rules for each cell: rules 43 and 44 for the left-hand side and rules 45 and 46 for the right-hand one.

Table 6 *Motion of a single locomotive through the fixed switch. To left: the locomotive arrives from the left-hand side. To right: it arrives from the right-hand side.*

	F	o	w	v _ℓ	i _ℓ	v _r	i _r		F	o	w	v _ℓ	i _ℓ	v _r	i _r
1	35	11	23	13	12	23	11	1	35	11	23	11	11	25	12
2	36	11	23	14	13	23	11	2	39	11	23	11	11	26	13
3	37	12	23	11	43	23	46	3	37	12	23	11	43	23	46
4	38	13	24	11	11	23	11	4	38	13	24	11	11	23	11
5	35	14	25	11	11	23	11	5	35	14	25	11	11	23	11
6	35	11	23	11	11	23	11	6	35	11	23	11	11	23	11
7	35	11	23	11	11	23	11	7	35	11	23	11	11	23	11

Table 7 Motion of two contiguous locomotives through the fixed switch. To left: the locomotives arrive from the left-hand side. To right: they arrive from the right-hand side.

	F	o	w	v _ℓ	i _ℓ	v _r	i _r		F	o	w	v _ℓ	i _ℓ	v _r	i _r
1	36	11	23	16	15	23	11	1	39	11	23	11	11	28	15
2	40	12	23	14	44	23	46	2	42	12	23	11	43	26	45
3	41	15	24	11	43	23	46	3	41	15	24	11	43	23	46
4	38	16	27	11	11	23	11	4	38	16	27	11	11	23	11
5	35	14	25	11	11	23	11	5	35	14	25	11	11	23	11
6	35	11	23	11	11	23	11	6	35	11	23	11	11	23	11
7	35	11	23	11	11	23	11	7	35	11	23	11	11	23	11

3.3 The round-about

In this subsection, we first look at the duplicator and then at the selector.

Duplicator

The study of the duplicator is illustrated by Figure 19 and the rules are given by Table 8. Also, Table 9 allows us to check the application of the rules and the working of the duplicator. For Table 9, note that \mathfrak{D} is the centre of the duplicator, that \mathfrak{i} is the entrance for the locomotive, that \mathfrak{i}_1 is the neighbour of \mathfrak{i} on the tracks leading to \mathfrak{D} , that \mathfrak{o} is the exit through which the two locomotives leave \mathfrak{D} and that they go on the tracks, first through \mathfrak{o}_1 and then through \mathfrak{o}_2 .

Note that the crossing of an element of the tracks or of a fixed switch by a single locomotive requires four steps exactly. If a locomotive is about to enter such a cell at time t , it just left the exit at time $t+4$. Now, when two consecutive locomotives cross the same elements, one more time is required. Here, we can see a similar situation: a single locomotive enters but two contiguous ones exit, so that one more step is needed but no more, as the pattern is reduced to a specific neighbouring of a white cell.

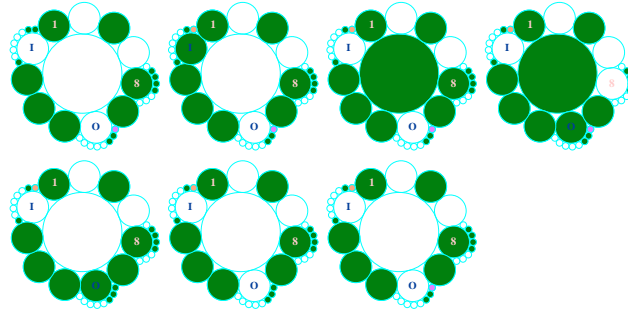


Figure 19 The motion of the locomotive through the duplicator. One locomotive enters and two ones leave the pattern.

Table 8 The rules for the duplicator. To left: the rules for the central cell. To right, the rules for neighbour $\mathfrak{8}$, \mathfrak{n}_8 .

the central cell		neighbour $\mathfrak{8}$	
	$\mathfrak{i} \quad \mathfrak{o} \quad \mathfrak{n}_8$		\mathfrak{D}
47	<u>WB</u> BBBBBWBW	52	BBBBBWBW
48	WB <u>BB</u> BBBWBW	53	BBBBBWBW
49	BBWBBBWBW	54	WB <u>BB</u> BBBWBW
50	BBWBBBWBW		
51	WBWBBBWBW		

The creation of the second locomotive is triggered by rule 50. It introduces a one step delay in the motion by keeping the central cell to be black.

Now, rule 50 might produce infinitely many locomotives. In order to reduce the creation of a new locomotive to a single one, neighbour **8** of the central cell flashes as the first locomotive entered **D** by rule 53 which makes it turn from black to white. Rule 54 makes neighbour **8** return to the black state at the following step. When rule 53 has been applied, so that neighbour **8** is white, rule 50 applied to the central cell makes the second locomotive leave the cell. Now, as shown by Tables 8 and 9, neighbour **8** is ruled by rule 52.

Table 9 *Motion of the locomotive through the duplicator. D is the central cell, i, o the entrance, exit, respectively for the locomotive.*

	i ₁	i	D	o	o ₁	o ₂	n ₈
1	25	24	47	23	11	11	52
2	26	25	48	23	11	11	52
3	23	26	49	24	11	11	53
4	23	26	50	27	12	11	54
5	23	23	51	28	15	12	52
6	23	23	47	26	16	15	52
7	23	23	47	23	14	13	52
8	23	23	47	23	11	11	52

Selector

As a consequence of the task assigned to the selector, its structure is more complex than what we have up to now studied. Figures 20 and 21 show us that several cells around the track used by the locomotive are involved in the working of the structure. Tables 11 and 12 show that thirteen cells are actively involved in the working of the selector. Also, Table 10 indicates that 43 new rules are needed. Tables 11 and 12 show that 12 rules of Table 2 are also involved. Note that globally the automaton makes use of 115 rules, so that the selector requires almost the half of it.

The thirteen cells involved in the working of the selector are **i**, the neighbour of **B** through which the locomotive(s) enter(s) the selector after crossing **i₁**, the neighbour of **i** on the tracks arriving to the selector: see Figure 20, last line, where the cells visited by the locomotive(s) are indicated in light colours. After **B** comes **A** from where the selection occurs, controlled by **c** and **D** which both can see **A** and **B** at the same time. From **A**, two locomotives exit, one through **o_r**, neighboured by **D**, the other through **o_ℓ** which is neighboured by **c**. The destruction of the superfluous locomotive requires to also examine two additional cells of the tracks on each side: **o_r¹** and **o_r²** on the right hand side, **o_ℓ¹** and **o_ℓ²** on the left hand side. Due to the number of neighbours required for **c**, another neighbour of **c**, namely **E**, is involved in this destruction process. Note that **E** is also a neighbour of the cells **o_ℓ**, **o_ℓ¹**

and \mathfrak{o}_ℓ^2 , while \mathfrak{c} is a neighbour of \mathfrak{o}_ℓ only.

Table 10 *The rules for the selector. The table gives the rules for the cells A, D, C and E. Also see Figures 20 and 21.*

A		i		C	
	CBD _{or} o _l		B i _l		AoE B
55	<u>WBWBWBWBWBWBWBWB</u>	62	<u>WBWBWBWBWBWBWBWB</u>	69	<u>BWBWBWBWBWBWBWBWB</u>
56	<u>WBWBWBWBWBWBWBWB</u>	63	<u>WBWBWBWBWBWBWBWB</u>	70	<u>BWBWBWBWBWBWBWBWB</u>
57	<u>BWBWBWBWBWBWBWBWB</u>	64	<u>BWBWBWBWBWBWBWBWB</u>	71	<u>BWBWBWBWBWBWBWBWB</u>
58	<u>WBWBWBWBWBWBWBWB</u>	65	<u>WBWBWBWBWBWBWBWB</u>	72	<u>WBWBWBWBWBWBWBWB</u>
	2 locomotives	66	<u>WBWBWBWBWBWBWBWB</u>	73	<u>BWBWBWBWBWBWBWBWB</u>
59	<u>BWBWBWBWBWBWBWBWB</u>		2 locomotives		2 locomotives
60	<u>WBWBWBWBWBWBWBWB</u>	67	<u>BWBWBWBWBWBWBWBWB</u>	74	<u>BWBWBWBWBWBWBWBWB</u>
	fork only:	68	<u>BWBWBWBWBWBWBWBWB</u>	75	<u>BWBWBWBWBWBWBWBWB</u>
61	<u>WBWBWBWBWBWBWBWB</u>			76	<u>BWBWBWBWBWBWBWBWB</u>
D		E		B	
	BA 21 _{or}		around E		
77	<u>BWBWBWBWBWBWBWBWB</u>	83	<u>BWBWBWBWBWBWBWBWB</u>	89	<u>WBWBWBWBWBWBWBWB</u>
78	<u>BWBWBWBWBWBWBWBWB</u>	84	<u>BWBWBWBWBWBWBWBWB</u>	90	<u>BWBWBWBWBWBWBWBWB</u>
79	<u>BWBWBWBWBWBWBWBWB</u>	85	<u>BWBWBWBWBWBWBWBWB</u>	91	<u>WBWBWBWBWBWBWBWB</u>
80	<u>BWBWBWBWBWBWBWBWB</u>		2 locomotives		2 locomotives
	2 locomotives	86	<u>BWBWBWBWBWBWBWBWB</u>	92	<u>BWBWBWBWBWBWBWBWB</u>
81	<u>BWBWBWBWBWBWBWBWB</u>	87	<u>BWBWBWBWBWBWBWBWB</u>	93	<u>BWBWBWBWBWBWBWBWB</u>
82	<u>WBWBWBWBWBWBWBWB</u>	88	<u>BWBWBWBWBWBWBWBWB</u>	94	<u>WBWBWBWBWBWBWBWB</u>

Table 11 *The scheduling of the crossing of the selector by a single locomotive.*

	i_1	i	B	A	o_r	o_r^1	o_r^2	o_ℓ	o_ℓ^1	o_ℓ^2	D	C	E
1	13	63	23	55	11	11	17	29	23	29	76	69	83
2	14	64	24	55	11	11	17	29	23	29	76	70	83
3	11	65	25	56	11	11	17	29	23	29	77	71	83
4	11	62	26	57	12	11	17	30	23	29	78	22	83
5	11	66	89	58	13	12	17	96	24	29	79	72	84
6	11	62	23	55	14	13	18	97	90	46	80	73	85
7	11	62	23	55	11	14	19	29	23	29	77	69	83
8	11	62	23	55	11	11	17	29	23	29	76	69	83

Figures 20 and 21 can be viewed as different zooms with respect to Figure 12 of Sub-section 2.2. Tables 11 and 12 allow us to follow how the rules are applied to the different cells constituting the selector.

Let us make the description more precise. The arrival by i_1 makes use of rules from Table 2 only. Now, i is not an ordinary element of the tracks. It has a specific configuration which is illustrated in Figure 20. Rules 62, 63, 64 and 65 and also 67 and 68 play the role of motion rules for the cell i . Rule 66 is used when the cell c is white. Then, the locomotive(s) arrive(s) at b . This cell is alike an element of the track with this restriction that two of its milestones may turn white at one moment exactly and only for that instant. So that the rules of Table 2 apply except when c or d is white. In the first case, rule 89 is applied. In the other one, it is rule 91. The cell a is ruled by rules 55 up to 60. Rule 55 manages the idle configuration. Rule 56 make the locomotive enter the cell and rule 57 kills it at the next

time. Rule 58 witnesses that each exit, the one close to c , the other close to d , is occupied by a locomotive. When there are two locomotives, rules 59 and 60 are applied instead of rules 57 and 58. Rule 61 is used only for the fork which has exactly the same neighbourhood as an idle cell A .

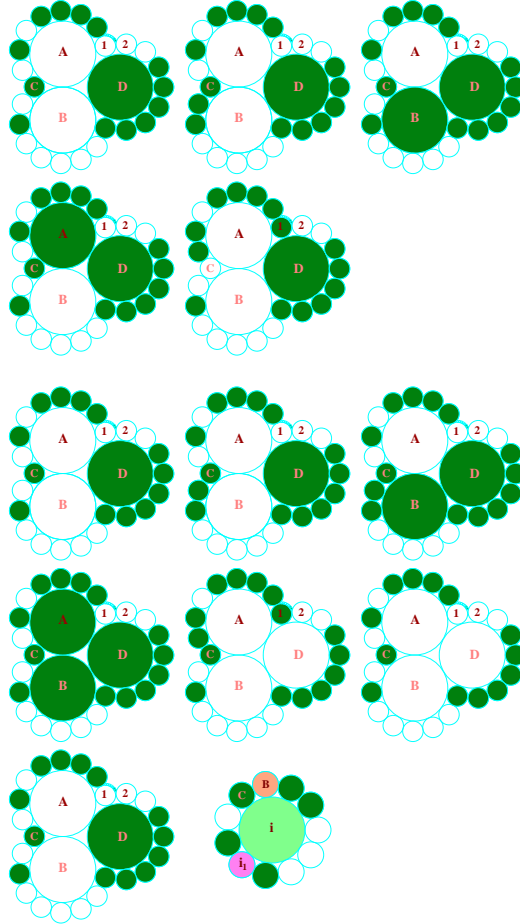


Figure 20 *The motion of the locomotive through the selector of a round-about. Focus on the cells A, B and D. On the last line of the figure, i and its neighbourhood.*

We can notice that the cells c and d are applied very different rules depending on the number of locomotives arriving at the selector. Rules 76 up to 80 are used when there is a single locomotive, witnessing the passage through the cells of the track which are in contact with d . Rule 81 detects

that two locomotives arrived and turn the state of the cell D to white, while rule 82 restores the black state at the next tick of the clock. Similarly, rules 69 up to 76 are used the cell C . It should be noticed that when there is a single locomotive in A , the configuration of the neighbours is that of a cell of the track in between two pivots when a locomotive is in the cell: this is why rule 22 is used to make the cell turn to white. When there are two locomotives, rules 74 up to 76 are used. Note the rules for E : rules 83 up to 85 when there is a single locomotive and rules 86 up to 88 when there are two of them.

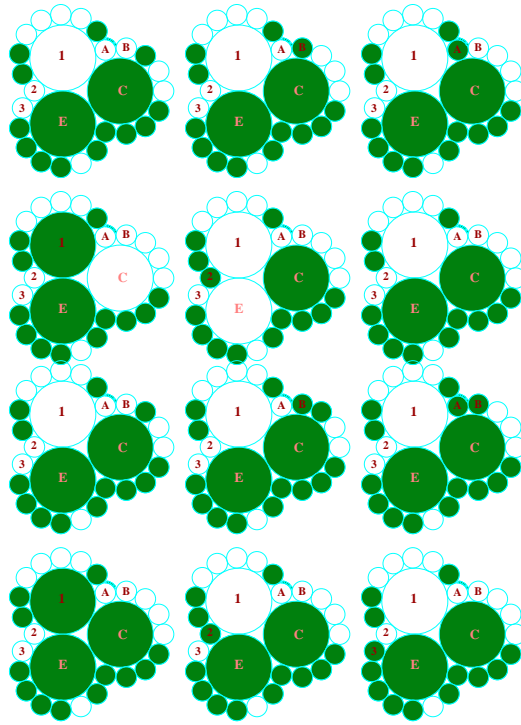


Figure 21 *The motion of the locomotive through the selector of a round-about. Focus on the cells C and E .*

The remaining rules of the table appear in the destruction of the superfluous locomotive: rule 89 for B when C , one of its milestones, momentarily vanishes. Rule 90 kills the locomotive which is at o_ℓ^1 as the cell E is white at this instant. Rule 91 is used for B when D , also one of its milestones, is white for two ticks of the clock. Rule 92 is used by o_r as its black but its milestone D is now white: and so, for the new time when D is still white,

rule 94 applies. Rules 93 and 94 are used for o_r^1 which, in this case remains white but has a missing milestone: \mathbf{D} . For a similar reason, rule 95 is used for o_r^2 as for it, \mathbf{D} is at another milestone. Eventually, rule 96 is used by o_ℓ when \mathbf{C} is white and rule 97 is used at the next time, \mathbf{E} being then white.

Table 12 *The scheduling of the crossing of the selector by two contiguous locomotives.*

	i_1	i	\mathbf{B}	\mathbf{A}	o_r	o_r^1	o_r^2	o_ℓ	o_ℓ^1	o_ℓ^2	\mathbf{D}	\mathbf{C}	\mathbf{E}
1	16	67	24	55	11	11	17	29	23	29	76	70	83
2	14	68	27	56	11	11	17	29	23	29	77	74	83
3	11	65	28	59	12	11	17	30	23	29	81	75	83
4	11	62	91	60	92	93	95	31	24	29	82	76	86
5	11	62	91	65	94	94	95	32	25	30	72	69	87
6	11	62	23	55	11	11	17	29	26	31	76	69	88
7	11	62	23	55	11	11	17	29	23	29	76	69	83
8	11	62	23	55	11	11	17	29	23	29	76	69	83

3.4 The active switches

From Section 2, we know that we have two active switches: the flip-flop and the part of the memory switch which is actively crossed by the locomotive. For these switches, we use two common structures: the fork and the controller. Figures 4 and 6 from Subsection 2.1 illustrate how these patterns are assembled in each case.

Fork

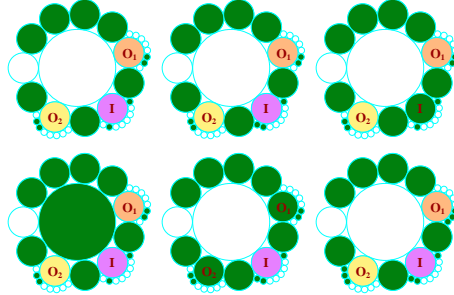


Figure 22 *The motion of the locomotive through the fork in the switches.*

The idle configuration of the fork is the same as that of the cell \mathbf{A} in the selector. This can be checked on Figure 22. The centre of the fork makes use of the rules 55, 56 and 57 from those used by the cell \mathbf{A} . As already mentioned it also makes use of rule 61. The slight differences with the rules applied to the cell \mathbf{A} of the selector come from the fact that in the neighbourhood of the centre of the fork, the black cells are invariant.

Table 13 *The scheduling of the crossing of the fork by a single locomotive.*

	i_1	i	t	o	o_r	o_ℓ	o_ℓ^1
1	19	24	55	11	17	11	17
2	20	25	56	11	17	11	17
3	17	26	57	12	17	12	17
4	17	23	61	13	18	13	18
5	17	23	55	14	19	14	19
6	17	23	55	11	17	11	17

As will be seen further again, the fork will also be used in the passive memory switch as already shown in Figure 7 from Subsection 2.1.

Controller

For checking the controller of the active switches, we just need Figure 14 and the explanations of Subsection 2.1. The rules are given by Table 14.

Table 14 *Rules for the controller used by the active switches.*

the cell t				the cell c			
C	o	i		t	s	t	s
98	<u>W</u>	<u>B</u>	<u>B</u>	<u>B</u>	<u>B</u>	<u>B</u>	<u>B</u>
				99	<u>W</u>	<u>B</u>	<u>B</u>
				100	<u>B</u>	<u>B</u>	<u>B</u>
				101	<u>W</u>	<u>B</u>	<u>B</u>
				102	<u>B</u>	<u>B</u>	<u>B</u>

From the right-hand side of Figure 14, when the cell c of the controller is white, the cell t looks like a cell of the tracks: its neighbourhood is exactly one of the neighbourhoods for cells which are elements of the tracks. Note the use of rule 61 for the cell c when the controller is white, leaving the locomotive cross the controller, see the lower half of Table 15.

Table 15 *The scheduling of the passage of the locomotive through the cell t , depending on whether the controller is black, upper table, or white, lower table.*

	i_1	i	t	o	o_r	o_r^1	C	s	s_1
1	13	12	97	11	11	11	100	26	11
2	14	13	98	11	11	11	100	26	11
3	11	11	97	11	11	11	100	26	11
4	11	11	97	11	11	11	100	26	11

	i_1	i	t	o	o_r	o_r^1	C	s	s_1
1	13	12	23	11	11	11	99	23	11
2	14	13	24	11	11	11	99	23	11
3	11	14	25	12	11	11	61	23	11
4	11	11	26	13	12	11	99	23	11
5	11	11	23	14	13	12	99	23	11
6	11	11	23	11	14	13	99	23	11
7	11	11	23	11	11	11	99	23	11

When the cell c is black, rule 97 manages the idle configuration of t and rule 98 prevents a locomotive to enter t when c is black. The two right-hand

side columns of Table 14 handle the behaviour of the cell c .

Table 16 *The scheduling of the change in the controller. Upper table: from unlocked to locked. Lower table: from locked to unlocked.*

	i_1	i	t	o	o_r	o_r^1	C	s	s_1
1	11	11	23	11	11	11	99	24	13
2	11	11	23	11	11	11	101	25	14
3	11	11	97	11	11	11	100	26	11
4	11	11	97	11	11	11	100	26	11
	i_1	i	t	o	o_r	o_r^1	C	s	s_1
1	11	11	97	11	11	11	100	72	13
2	11	11	97	11	11	11	102	28	14
3	11	11	23	11	11	11	99	23	11
4	11	11	23	11	11	11	99	23	11

Rules 99 and 100 manage the idle configuration in which c may be either white or black but permanently in the same state. Rules 101 and 102 handle the change of state of c which is triggered by the arrival of a locomotive through its neighbour 11. Note that both rules have exactly the same context as in both cases, the neighbourhood of c is the same.

3.4.1 The passive memory switch

From Subsection 2.1, we know that the new pattern involved in the passive memory switch is the structure we called the sensor. The structure is illustrated by Figure 15.

Sensor

As in the case of the controller, if the cell c is white, the cell t behaves as a cell of the track as it has one of the defined neighbourhoods for the elements of the tracks. However, and it is one of the differences with what happens in the controller, when c is black, the locomotive must nevertheless cross the cell t . So that in this case, we have new motion rules given by rules 103 up to 106, that one included.

This time, the cell c has two auxiliary cells s and e . Rules 114 and 115 handle the case of the idle configuration for s , no matter which is the state of c . When the cell c is white, if a locomotive runs through t , c remains white as this is the case of a passage through the selected track: this is checked by rule 107. When c is black, this means that the track is not selected. The passage of the locomotive requires that c changes to white and that it will keep the state white until a signal of a change comes through the cell e . The change from black to white for c is controlled by rule 108. But at the same

time, s must flash from white to black and back to white through rules 56 and 68: look at Table 18. When s has flashed, note the locomotive moving from v_S and v_S^1 : this locomotive will reach the other sensor of the switch to there make the cell E flash in order to turn the cell c from white to black.

Table 17 *Rules for the sensor used by the passive memory switch.*

cell t			cell c			cell e			cell s		
C o i			tS E			C i			Ct o		
103	W	B	B	B	B	B	B	B	W	W	W
104	W	B	B	B	B	B	B	B	W	W	B
105	B	B	B	B	B	B	B	B	W	W	W
106	W	B	B	B	B	B	B	B	W	W	B
107	W	B	B	B	B	B	B	B	W	W	W
108	B	B	B	B	B	B	B	B	W	W	W
109	W	B	B	B	B	B	B	B	W	W	B
110	W	B	B	B	B	B	B	B	W	W	W
111	W	B	B	B	B	B	B	B	W	W	W
112	W	B	B	B	B	B	B	B	W	W	B
113	B	B	B	B	B	B	B	B	W	W	W

Let us look at that: the auxiliary locomotive arrives to v_E^1 and then to v_E . As can be seen in Table 18, rule 112 makes E turn from white to black and then rule 113 makes in turn back to white. But when c can see that E is black, it turns from white to black thanks to rule 109. Note that for E , rules 110 and 111 manage its idle configuration whatever the state of c .

Table 18 *The scheduling of the passage of the locomotive controlled by the sensor.*

	i_1	i	t	o	o_r	C	E	S	v_E	v_E^1	v_S	v_E^1
1	13	12	103	11	11	79	111	114	11	11	11	11
2	14	13	104	11	11	79	111	114	11	11	11	11
3	11	14	105	12	11	108	111	56	11	11	11	11
4	11	11	106	13	12	58	110	68	11	11	12	11
5	11	11	29	14	13	82	110	115	11	11	13	12
6	11	11	29	11	11	82	110	65	11	11	14	13
	i_1	i	t	o	o_r	C	E	S	v_E	v_E^1	v_S	v_E^1
1	13	12	29	11	11	82	110	65	11	11	11	11
2	14	13	30	11	11	82	110	65	11	11	11	11
3	11	14	31	12	11	107	110	55	11	11	11	11
4	11	11	32	13	12	82	110	65	11	11	11	11
5	11	11	29	14	13	82	110	65	11	11	11	11
6	11	11	29	11	11	82	110	65	11	11	11	11
	i_1	i	t	o	o_r	C	E	S	v_E	v_E^1	v_S	v_E^1
1	11	11	29	11	11	82	110	65	12	13	11	11
2	11	11	29	11	11	82	112	65	13	14	11	11
3	11	11	29	11	11	109	113	65	14	11	11	11
4	11	11	103	11	11	79	111	114	11	11	11	11
5	11	11	103	11	11	79	111	114	11	11	11	11
6	11	11	103	11	11	79	111	114	11	11	11	11

With the help of the figures, of the tables for the rules and of the schedule tables we have proved the following result:

Theorem 1 *There is a weakly universal cellular automaton on the tiling $\{11, 3\}$ which is planar and rotation invariant.*

We remind the reader that with *planar*, we mean that the set of cells which are crossed by the locomotive(s) contains infinitely many cycles.

3.5 Concluding remarks

It can be asked whether it is possible or not to lower the number of neighbours in order to get a planar rotation invariant weakly universal cellular automaton in a tiling $\{p, 3\}$ or $\{p, 4\}$.

A first remark on the number of rules.

The tables we have displayed for the rules indicate 115 of them. However, there might be more rules: this depends on what we call the program of the cellular automaton. We should notice that these 115 rules are rotationally independent: none of them can be obtained from another one by a circular permutation on the neighbour's states. Accordingly, if we consider that the program of a rotation invariant cellular automaton should contain all rotated forms of the same rule, there should be much more rules: roughly 115×11 . However, this is not exactly true. If we consider the motion rules for the tracks, then it is likely that any rotated form will be met in the construction of the tracks needed for the simulation of a register machine. However, for the cells concerning a structure, this may be not the case. It may be arranged that all configurations make use of the same rules which are the ones used in the tables of the paper, or that we have only to take two or three rotated forms of the same rules but not all of them.

Table 19 gives us the list of the rules which occur in two different settings. As can be checked, for the time and the cell to which they apply, the configurations of the neighbours and the state of the cell is the same.

It should be noticed that the same cell may appear several times in this table. As an example, the cell \mathfrak{t} of the fork appears four times in the table. It appears twice when the fork is idle, also when the locomotive is about to enter \mathfrak{t} and when two locomotives leave the fork. The neighbourhood is that of the cell \mathfrak{a} of the selector when it is idle, when the locomotive is in \mathfrak{i} in the selector, when the locomotive is in \mathfrak{a} in the selector and for the cell \mathfrak{c} of the controller when the locomotive there is in \mathfrak{t} . In rules 57 and 61 which both apply to the fork and something else, the context differ by the fact that \mathfrak{o}_r is white in rule 57 but black in rule 61. We can see that the neighbourhood of the cell is rotated in one rule with respect to what it is in the other rule.

Note also that rules 79 and 82 have the same context, but the state of the cell is different. In this case each rule also applies to two different cells. However, there are several cases of different rules having the same context. This is in particular the case for motion rules applied to the same cell and

It is possible that the rather big number of rules applied in different contexts indicates that it must be difficult to lower the number of sides of a tile in a tiling $\{p, 3\}$ in order to obtain a weakly universal cellular automaton which would be rotation invariant and which would be planar. This seems difficult at least using the railway model. I say it might be difficult, I cannot say it is impossible.

46	$\begin{array}{c} \text{F i o} \\ \underline{\text{WBBWBWBWBWWWW}} \\ \text{o Ei} \end{array}$	fix: $i_r, 3$	sel: $o_\ell^2, 6_1$
55	$\begin{array}{c} \text{CBD} o_r \quad o_\ell \\ \underline{\text{WBWBWBWBWBWBW}} \\ \text{i } o_r \quad o_\ell \\ \text{Ct} \quad v_S \end{array}$	sel: A, 0	for: t, 0 sen: s, 3
56	$\begin{array}{c} \text{CBD} o_r \quad o_\ell \\ \underline{\text{WBWBWBWBWBWB}} \\ \text{i } o_r \quad o_\ell \\ \text{Ct} \quad v_S \end{array}$	sel: A, $2_2, 3_1$	for t, 2 sen: s, 3
57	$\begin{array}{c} \text{CBD} o_r \quad o_\ell \\ \underline{\text{BWBWBWBWBWBW}} \\ \text{i } o_r \quad o_\ell \end{array}$	sel: A, 4_1	for: t, 3
58	$\begin{array}{c} \text{CBD} o_r \quad o_\ell \\ \underline{\text{WWWBWBWBWBW}} \\ \text{tSE} \end{array}$	sel: A, 5	sen: c, 0
61	$\begin{array}{c} o_r \quad o_\ell \text{ i} \\ \underline{\text{WBWBWBWBWBW}} \\ \text{t} \quad \text{s} \end{array}$	for: t, 4	con: c, 3
65	$\begin{array}{c} \text{B} \quad \text{i}_1 \\ \underline{\text{WBWBWBWBWBWW}} \\ v_S \text{Ct} \end{array}$	sel: i, 3; A, 5	sen: s, 0
68	$\begin{array}{c} \text{B} \quad \text{i}_1 \\ \underline{\text{BWBWBWBWBWBWW}} \\ v_S \text{Ct} \end{array}$	sel: i, 2	sen: s, 4
72	$\begin{array}{c} \text{AoE} \quad \text{B} \\ \underline{\text{WBWBWBWBWBWB}} \\ \text{C } s_1 \end{array}$	sel: C, 5_1 ; D, 5_2	con: s, 0
79	$\begin{array}{c} \text{BA} \quad 2\text{i} o_r \\ \underline{\text{BWBWBWBWBWB}} \\ \text{s} \quad \text{t} \end{array}$	sel: D, 5_1	sen: c, 0
82	$\begin{array}{c} \text{BA} \quad 2\text{i} o_r \\ \underline{\text{WBWBWBWBWBW}} \\ \text{s} \quad \text{t} \end{array}$	sel: D, 4_2	sen: c, 0
97	$\begin{array}{c} v\text{ECA} \\ \underline{\text{WBWBWBWBWBWW}} \\ \text{C } o \text{ i} \end{array}$	sel: $o_\ell, 6_1$	con: t, 0

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